

Relation between (e, e') sum rules in ${}^6, {}^7\text{Li}$ and ${}^4\text{He}$ nuclei. Experiment and cluster model

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(Dated: July 26, 2016)

The sums over (e, e') spectra of ${}^6\text{Li}$ and ${}^7\text{Li}$ nuclei which correspond to the longitudinal sum rule are studied. It is suggested that due to the cluster structure of the lithium isotopes these sums may approximately be expressed in terms of such a sum pertaining to the α -particle. Calculation of these sums is performed in the framework of cluster models with antisymmetrization done with respect to all the nucleons. At momentum transfers higher than 0.8 fm^{-1} the relations expressing the $A=6$ or 7 sum in terms of the $A=4$ sum prove to be valid with rather high accuracy. In the region of momentum transfers around 1 fm^{-1} the longitudinal correlation functions of ${}^6\text{Li}$ and ${}^7\text{Li}$ nuclei are found to be close to that of the α -particle. Basing on this, the difference between the q values at which the high- q limit of the inelastic sum rule is reached in the ${}^6, {}^7\text{Li}$ cases and the ${}^4\text{He}$ case is explained. The experimental longitudinal sums in the range between 0.450 and 1.625 fm^{-1} are employed to perform comparison with the theoretical sum rule calculated in the framework of cluster models. Out of the experimental sums, those in the range between 0.750 and 1.000 fm^{-1} in the ${}^6\text{Li}$ case and between 0.750 and 1.125 fm^{-1} in the ${}^7\text{Li}$ case are obtained in the present work. In the ${}^6\text{Li}$ case a complete agreement between experiment and the calculated sum rule is found while in the ${}^7\text{Li}$ case an agreement only at a qualitative level is observed.

I. INTRODUCTION

The electronuclear sum rules, see e.g. [1], determine two-nucleon correlation functions. In Sect. 2 it is argued that at momentum transfers around 1 fm^{-1} the longitudinal correlation functions of ${}^6\text{Li}$ and ${}^7\text{Li}$ nuclei are close to that of the α -particle. This originates from the cluster structure, see e.g [2], of the lithium isotopes. It is argued that at momentum transfers higher than 0.8 fm^{-1} the sums over (e, e') spectra of ${}^6\text{Li}$ and ${}^7\text{Li}$ nuclei which correspond to the longitudinal sum rule may be expressed rather accurately in terms of such a sum pertaining to the α -particle. Calculation of these sums is performed in the framework of cluster models with antisymmetrization done with respect to all the nucleons.

In Sect. 3 experimental studies at Kharkov Institute of Physics and Technology (KIPT) of electronuclear reactions with the lithium isotopes are outlined. Basing on the extracted response functions, experimental sums corresponding to the longitudinal sum rule have been obtained in the ${}^6\text{Li}$ case at various momentum transfers lying in the range between 0.450 fm^{-1} and 1.450 fm^{-1} [3] and between 1.125 fm^{-1} and 1.625 fm^{-1} [4]. In the ${}^7\text{Li}$ case, such sums have been obtained at momentum transfers in the range between 1.250 fm^{-1} and 1.625 fm^{-1} [5]. In the present work, the experimental sums have been obtained at several momentum transfers in the range between 0.750 fm^{-1} and 1.000 fm^{-1} in the ${}^6\text{Li}$ case and

between 0.750 fm^{-1} and 1.125 fm^{-1} in the ${}^7\text{Li}$ case. For both nuclei the experimental sums are compared with the theoretical sum rule calculated in cluster models.¹

The notation below is as follows. Let $R_L(q, \omega)$ and $R_T(q, \omega)$ be the longitudinal and transverse response functions entering the (e, e') cross section. For transitions to continuum one has

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_{\text{Mott}} \left[\frac{Q^4}{q^4} R_L(q, \omega) + \left(\frac{Q^2}{2q^2} + \tan^2 \frac{\theta}{2} \right) R_T(q, \omega) \right]. \quad (1)$$

Here q and ω are the momentum and energy transfer from electron to the nucleus, and $Q^2 = q^2 - \omega^2$. For transitions to discrete levels $R_{L,T}$ are defined in the same way provided that the left-hand side is replaced with $f_{\text{rec}} d\sigma/d\Omega$ where f_{rec} is the recoil factor.

We shall consider the longitudinal sums

$$S_L(q) = \sum_n \frac{R_L(q, \omega_n)}{[\tilde{G}_E^p(Q_n)]^2} + \int_{\omega_{\text{thr}}}^{\infty} d\omega \frac{R_L(q, \omega)}{[\tilde{G}_E^p(Q)]^2} \quad (2)$$

where the summation goes over discrete levels and includes the elastic contribution. Here and below the notation of the type $\tilde{G}_E^{p,n}(Q) = G_E^{p,n}(Q)[1 + Q^2/(4m^2)]^{-1/2}$ is used, the $G_E^{p,n}$ notation stands for the electric Sachs proton and neutron form factors, m is the nucleon mass, and $Q_n^2 = q^2 - \omega_n^2$.

We employ the one-body charge transition operator.

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¹ Sect. 2 is written by V.D.E. and Sect. 3 is written by I.S.T. and A.Yu.B.

One then has the well-known longitudinal sum rule

$$S_L(q) = Z + \frac{1}{2J+1} \sum_{M=-J}^J \langle \Psi_M | \sum_{k \neq l}^A \hat{e}_k \hat{e}_l e^{i\mathbf{q} \cdot (\mathbf{r}_k - \mathbf{r}_l)} | \Psi_M \rangle. \quad (3)$$

Here Ψ_M denotes the ground state with the spin J and its projection M , and

$$\hat{e}_k = 1/2(1 + \tau_{zk}) + 1/2(1 - \tau_{zk})G_E^n(q)/G_E^p(q)$$

τ_{zk} being the nucleon isospin projection. In Eq. (3) and below the squared neutron form factor and also the corrections to G_E^n/G_E^p of the order of $(q/m)^2$ are neglected. We shall consider only moderate q values which justifies [6] the disregarding of two-body charge operators.

Comparison with experiment is done for inelastic (e, e') sums so that the elastic contribution is subtracted from the total sum (3). In addition, the normalization to unity in the high q limit is adopted. Thus, the quantities that are compared with experiment are the inelastic sums

$$(S_L)_{\text{inel}}(q) = Z^{-1} \{ S_L(q) - [ZF_{\text{el}}(q)/\tilde{G}_E^p(q)]^2 \} \\ = 1 + f_{\text{corr}}(q) - Z[F_{\text{el}}(q)/\tilde{G}_E^p(q)]^2 \quad (4)$$

where the correlation contribution f_{corr} corresponds to the second term in Eq. (3) and F_{el} is the charge elastic form factor.

In experiment of Ref. [4] it was found that in the ${}^6\text{Li}$ nucleus case the $(S_L)_{\text{inel}}$ sum approaches its high q limit faster than for other nuclei. In this connection let us compare the cases of ${}^6\text{Li}$ and ${}^4\text{He}$ nuclei. The ${}^4\text{He}$ nucleus behaves like many other nuclei in this respect. Therefore, an explanation of the difference between the two cases would provide, to some degree, an explanation of the mentioned feature. In general, the velocity with which the inelastic sum of Eq. (4) approaches its high q limit depends on the velocities of the approaching to zero of the correlation and the elastic form factor contributions. In the ${}^4\text{He}$ case, the second of these velocities is lower. Indeed, it is seen from Table 1 in Ref. [7] that the inelastic $(S_L)_{\text{inel}}$ sum of ${}^4\text{He}$ approaches the limit considerably slower than the total S_L sum does. As a result, the velocity with which the $(S_L)_{\text{inel}}$ sum approaches its high q limit is determined by the elastic form factor contribution *i.e.* by the size of ${}^4\text{He}$.

And, as argued below, in the ${}^6\text{Li}$ case the correlation contribution to the inelastic sum (4) behaves rather similar to that of ${}^4\text{He}$ as q increases. This behavior is due to the contribution of correlations between protons belonging to the ${}^4\text{He}$ cluster that is a constituent of ${}^6\text{Li}$. On the contrary, the absolute value of the contribution of the elastic form factor to the sum (4) decreases considerably faster than in the ${}^4\text{He}$ case. This is because of the larger size of ${}^6\text{Li}$. As a result, the $(S_L)_{\text{inel}}$ sum approaches its high- q limit faster than in the ${}^4\text{He}$ case. This feature is strengthened by the fact that in the ${}^6\text{Li}$ case the correlation contribution and the contribution of the elastic form factor occur to cancel each other to a considerable

degree in the corresponding q region. The said above applies equally to ${}^7\text{Li}$ which is a clustered nucleus of a considerable size as well.

II. SUM RULES AND CLUSTERING

The sum rule (3) includes the single-particle term, *i.e.* Z , and the correlation term. Let us consider a limiting case of a clustered nucleus with average distances between clusters much larger than sizes of the clusters. In such a case, at moderate q values, a predominant contribution to the correlation term comes from regions in the configuration space where clusters do not overlap (see also the reasoning below). Hence, cluster models are sufficient to calculate this term since they describe properly such regions. Although such models include the operator of antisymmetrization with respect to all the nucleons, at the above formulated condition the overlap of wave function components which differ in distributions of nucleons over clusters is not significant. Therefore, the simple cluster model without antisymmetrization may be employed.

At these conditions, the following can be suggested. Let r be the effective correlation distance for a pair of protons belonging to the same cluster, such that the corresponding correlation contribution in Eq. (3) becomes negligible when qr is large compared with unity. Let R be a typical distance between clusters or between protons belonging to different clusters. Taking into account that the role of the region of an overlap between clusters is small one may think that the correlation distance for protons belonging to different clusters is close to R . Correlations between such protons cease to play a substantial role when qR becomes large compared with unity.

The correlation distance r anyway cannot exceed the size of a cluster. Thus $r \ll R$. Therefore, there exists a range of q values at which correlations between protons belonging to the same cluster are not negligible in Eq. (3) while those between protons belonging to different clusters are negligible. In this regime, the correlation contribution in Eq. (3) can be well represented by the sum of such contributions pertaining to the constituent clusters.

This relation may deteriorate not only at low q values but also at sufficiently high q values. This is due to antisymmetrization effects disregarded above. Nevertheless, the following may be stated. If one adds Z to this relation then it turns to a relation between the longitudinal sums of a clustered nucleus and of its constituent clusters. And the latter relation remains approximately valid at mentioned higher q values since correlation contributions are small at these values.

Suppose that the above picture is applicable in the case of the lithium isotopes. Consider the above mentioned range of q values for which qR is large compared with unity. Then, at such q values, the following relations can

be suggested. In the ${}^6\text{Li}$ case,

$$S_L^{6\text{Li}}(q) \simeq S_L^{4\text{He}}(q) + 1. \quad (5)$$

In the ${}^7\text{Li}$ case,

$$S_L^{7\text{Li}}(q) \simeq S_L^{4\text{He}}(q) + S_L^{3\text{H}}(q). \quad (6)$$

(One may think that a similar relation is valid also in the case of ${}^9\text{Be}$ nucleus.)

The cluster model description of the lithium isotopes is used below both to estimate the accuracy of these relations and to calculate the $S_L^{6,7\text{Li}}$ sums. Let us start from the case of ${}^7\text{Li}$ that is a two-cluster nucleus. First, let us proceed in the framework of the cluster model without antisymmetrization. In this case the ground state wave function of ${}^7\text{Li}$ is

$$\Psi({}^7\text{Li}) = \psi_\alpha r_{\text{rel}} f(r_{\text{rel}}) [\psi_t Y_1(\mathbf{r}_{\text{rel}}/r_{\text{rel}})]_{J=3/2, M} \quad (7)$$

where \mathbf{r}_{rel} is the distance between the centers of mass of the two clusters, $r_{\text{rel}} f(r_{\text{rel}})$ describes the cluster relative motion, ψ_α is the wave function of the ${}^4\text{He}$ cluster, and ψ_t is the wave function of the ${}^3\text{H}$ cluster coupled with the spherical harmonics to the total momentum. The correlation term in Eq. (3) consists of three contributions, the first one corresponds to both nucleons with the k and l numbers belonging to the α -particle, the second one to both of them belonging to the tritium, and the third one to the case when a nucleon belongs to the α -particle and another nucleon to the tritium. The first of these contributions is equal to that of ${}^4\text{He}$. The second one is equal to that of ${}^3\text{H}$ which is seen if one takes into account a simple property of Clebsh-Gordan coefficients. Let us denote δ the third mentioned contribution. To calculate it we write $\mathbf{r}_k - \mathbf{r}_l$ as $\bar{\mathbf{r}}_k - \bar{\mathbf{r}}_l + \mathbf{r}_{\text{rel}}$ where $\bar{\mathbf{r}}_k$ and $\bar{\mathbf{r}}_l$ are the nucleon positions with respect to the centers of mass of the clusters. One obtains that this contribution is as follows,

$$\delta = 4[\tilde{G}_E^p(q)]^{-2} F_{\text{el}}^{4\text{He}}(q) F_{\text{el}}^{3\text{H}}(q) \int_0^\infty dr_{\text{rel}} r_{\text{rel}}^4 f^2(r_{\text{rel}}) \frac{\sin qr_{\text{rel}}}{qr_{\text{rel}}} \quad (8)$$

where $F_{\text{el}}^{4\text{He}}$ and $F_{\text{el}}^{3\text{H}}$ are the elastic scattering form factors. The fact that only the $l = 0$ multipole, *i.e.* a scalar operator, contributes to the elastic form factor of tritium and a simple property of Clebsh-Gordan coefficients are taken into account to get this expression.

We shall use the $f(r_{\text{rel}})$ function from Ref. [8]. It reproduces accurately all the available scattering and photodisintegration data on various processes involving ${}^7\text{Li}$. The last factor in Eq. (8) becomes very small when qR reaches values large compared to unity, R being the range of the relative motion function. To estimate the corresponding q values let us mention that the rms value pertaining to the function $r f(r)$ is 3.4 fm. This value is derived from the values of charge radii of ${}^7\text{Li}$, α -particle, and tritium. (The $f(r)$ we use leads to the value of 3.7 fm.)

TABLE I: The calculated ${}^7\text{Li}$ longitudinal sum. Its values without antisymmetrization with respect to nucleons belonging to different clusters are $S_L^{4\text{He}} + S_L^{3\text{H}} + \delta$. Its values when antisymmetrization is done are presented in the fourth column. In the fifth column somewhat improved values (see the text) are listed.

q, fm^{-1}	$S_L^{4\text{He}} + S_L^{3\text{H}}$	δ	a) $S_L^{7\text{Li}}$	b) $S_L^{7\text{Li}}$
0.5	4.60	1.96	6.50	6.58
0.6	4.46	1.41	5.81	5.87
0.7	4.31	0.95	5.19	5.22
0.8	4.15	0.59	4.66	4.68
0.9	3.99	0.34	4.23	4.24
1.0	3.84	0.18	3.90	3.89
1.1	3.69	0.085	3.66	3.64
1.2	3.56	0.040	3.47	3.45
1.3	3.44	0.023	3.34	3.32
1.4	3.34	0.022	3.25	3.22
1.5	3.25	0.025	3.18	3.15

Passing to the net longitudinal sums, one then may write $S_L^{7\text{Li}} = S_L^{4\text{He}} + S_L^{3\text{H}} + \delta$ in the present no-antisymmetrization case. The contributions $S_L^{4\text{He}} + S_L^{3\text{H}}$ and δ obtained are shown in Table 1 in the second and third column, respectively. The $S_L^{4\text{He}}(q)$ and $S_L^{3\text{H}}(q)$ sums are calculated with the help of the model-independent relations [7, 9] involving experimental elastic form factors of ${}^4\text{He}$, ${}^3\text{He}$, and ${}^3\text{H}$. These relations have been shown there to be accurate at $q \leq 1.5 \text{ fm}^{-1}$. In the ${}^4\text{He}$ case

$$S_L^{4\text{He}}(q) = 2 + \left[2 + 8 \frac{G_E^n(q)}{G_E^p(q)} \right] \frac{F_{\text{el}}^{4\text{He}}(\sqrt{8/3}q)}{\tilde{G}_E^p(\sqrt{8/3}q) + \tilde{G}_E^n(\sqrt{8/3}q)} \quad (9)$$

while $S_L^{3\text{H}}(q)$ differs from unity merely by a term proportional to $G_E^n(q)/G_E^p(q)$. In what follows the elastic form factors of ${}^4\text{He}$ and ${}^3\text{He}$ from Refs. [10, 11], that of ${}^3\text{H}$ from Ref. [12] (fit d), and the nucleon form factors from Ref. [13] are used.

It is seen that the δ contribution is, indeed, relatively small at $q \geq 0.8 \text{ fm}^{-1}$. At the same time, since the $S_L^{4\text{He}}(q) + S_L^{3\text{H}}(q)$ quantity differs sizably from the $Z = 3$ limit at corresponding q values, the contribution to the sum of the correlation term itself is considerable. Thus the relation (6) is rather accurate.

One also sees that the decrease of δ at higher q values is not monotonic. This is probably related to the fact that the relative motion function $f(r)$ has a "shell-model" node at $r \simeq 1.95 \text{ fm}$ that is in the region where the clusters overlap. Due to this, in addition to the larger correlation distance R there exists a smaller correlation distance r_0 related to the node. This r_0 distance is such that qr_0 is not so large as compared to unity. Mainly the r distances smaller than r_0 contribute to δ at higher q values. This contribution is small since the contribution

to the normalization integral from the distances below the node is 17% only.

Now, let us estimate the antisymmetrization effects at calculating the correlation function. The Ψ_M state in Eq. (3) is normalized to unity. Let us write it as $\Psi_M = \langle \mathcal{A}\psi_M | \mathcal{A}\psi_M \rangle^{-1/2} \mathcal{A}\psi_M$ where ψ_M is a clustered state corresponding to Eq. (7) and \mathcal{A} is the antisymmetrization operator. Below the ψ_M notation stands for the corresponding wave function. In the calculation below, wave functions of the clusters are chosen to be the products of spatial and spin-isospin functions. Then ψ_M is of the structure $\psi_M = [\varphi_{L=1}\chi_{STM_T}]_{J=3/2,M}$ where $\varphi_{L=1,M_L}$ are the spatial factors with given projections of the orbital momentum and $\chi_{SM_S TM_T}$ are the spin-isospin factors. The $\varphi_{L=1,M_L}$ factors consist of the spatial components of wave functions of the clusters and the function of their relative motion. The spin-isospin factors consist of spin-isospin functions of the clusters and correspond to given values of spin and isospin and of their projections.

The spatial components of wave functions of the clusters we employ are symmetric with respect to nucleon permutations. They are normalized Gaussians of the form $\text{const} \cdot \exp[-\lambda \sum_{k=1}^A (\mathbf{r}_k - \mathbf{R}_{cm})^2]$ where \mathbf{R}_{cm} is the center of mass variable of a cluster, and $A = 3$ or 4 . The λ parameters are chosen to fit rms matter radii of the clusters. The spin-isospin components of these wave functions are antisymmetric with respect to nucleon permutations. The cluster relative motion function is taken the same as above. It has the form [8] of a sum of Gaussians.

For brevity let us denote φ_{M_L} and χ the above defined $\varphi_{L=1,M_L}$ and $\chi_{SM_S TM_T}$ factors. Since the operator in Eq. (3) does not depend on spin variables, averaging over projections of the total spin there turns to averaging over projections of the orbital momentum. Taking also into account that $\mathcal{A}^2 = \text{const} \cdot \mathcal{A}$ one may write the correlation term in Eq. (3) in the form

$$\frac{S_L(q) - Z}{\sum_{k \neq l} \sum_{M_L=-1}^1 \langle \varphi_{M_L} \chi | \hat{e}_k \hat{e}_l e^{i\mathbf{q} \cdot (\mathbf{r}_k - \mathbf{r}_l)} \mathcal{A} | \varphi_{M_L} \chi \rangle} \quad (10)$$

Suppose that before antisymmetrization the nucleons with the numbers from one to four belong to the α -particle and those with the numbers from five to seven belong to the tritium. As a consequence of the fact that the alpha-particle and the tritium states are already antisymmetric with respect to nucleon permutations, only the inter-cluster antisymmetrization is to be done. Furthermore, one can see in this connection that the operator $1 - 12(\hat{15}) + 18(\hat{15})(\hat{26}) - 4(\hat{15})(\hat{26})(\hat{37})$, where (\hat{ij}) is a particle transposition, may be put in Eq. (10) in place of the total \mathcal{A} antisymmetrizer with no change in the result.

Let us comment on the calculation of the spatial factors in the nominator in Eq. (10), for example. Consider there a single term in the sum over nucleons and one of the four terms with particle permutations. The

corresponding contribution is an integral over 18 Jacobi variables whose integrand includes a Gaussian times a polynomial. As noted in Ref. [14] such type integrals can be done analytically. In the present calculation, after inserting the additional integration over the normalized Gaussian wave functions of the total center of mass, the arising total wave function is rewritten in terms of the position vectors of separate particles. This makes simple the action of particle permutations. The arising expression is a sum like $\sum_{m \leq n} b_{mn} I_{mn}^{kl}$ where b_{mn} are constants and I_{mn}^{kl} are integrals of the form

$$I_{mn}^{kl} = \int d\mathbf{r}_1 \dots d\mathbf{r}_N (\mathbf{r}_m \cdot \mathbf{r}_n) \times \exp \left[- \sum_{s,t=1}^N a_{st} (\mathbf{r}_s \cdot \mathbf{r}_t) + i\mathbf{q} \cdot (\mathbf{r}_k - \mathbf{r}_l) \right] \quad (11)$$

at $N = 7$ and $a_{st} = a_{ts}$.

The integrals (11) are calculated as follows. As is known

$$\int dx_1 \dots dx_N \exp \left(- \sum_{s,t=1}^N a_{st} x_s x_t + \sum_{j=1}^N y_j x_j \right) = \frac{\pi^{N/2}}{\Delta^{1/2}} \exp \left[\frac{1}{4\Delta} \sum_{s,t=1}^N y_s y_t A_{st} \right]. \quad (12)$$

Here $\Delta = \det(a_{st})$ and A_{st} is the algebraic adjunct of the a_{st} element in the Δ determinant. To calculate the integral whose integrand differs from that in Eq. (12) by the $x_m x_n$ factor we make a replacement there of the type of $y_i \rightarrow y_i + \lambda \delta_{mi} + \nu \delta_{ni}$, calculate derivatives with respect to λ and ν , and then put λ and ν equal to zero. This leads to the following expression for the quantity (11),

$$I_{mn}^{kl} = \frac{\pi^{3N/2} [6A_{mn}\Delta - q^2(A_{mk} - A_{ml})(A_{nk} - A_{nl})]}{4\Delta^{7/2}} \times \exp \left[- \frac{q^2}{4\Delta} (A_{kk} + A_{ll} - 2A_{kl}) \right] \quad (13)$$

($A_{ij} = A_{ji}$.)

At $q = 0$ the correlation term in Eq. (3) should equal $Z(Z-1) = 6$. This is used as a test of the whole calculation. (To get this with high accuracy the form factors from [13] at $q = 0$ are to be corrected.) And the q dependent quantities (11) are calculated both using Eq. (13) and an alternative analytical expression obtained via differentiating the Eq. (12) type equalities with respect to a_{st} .

The results of the calculation are presented in the fourth column in Table 1. In order to check and somewhat improve them let us consider separately the "direct" term *i.e.* the contribution that arises when one replaces the \mathcal{A} operator with unity in Eq. (10). Let us replace this contribution with that calculated as the sum of the second and third columns in the Table. This corresponds to use of the true wave functions of the clusters in the

direct term in place of those employed above. This is because experimental data on form factors of the ${}^4\text{He}$ and ${}^3\text{H}$ nuclei are employed above to calculate the contribution $S_L^{4\text{He}} + S_L^{3\text{H}}$ and the contribution of Eq. (8) instead of theoretical wave functions of ${}^4\text{He}$ and ${}^3\text{H}$. The results obtained in this way are presented in the fifth column in the Table. The change in the results is not significant.

The values of the $S_L^{7\text{Li}}$ sum without antisymmetrization, *i.e.* $S_L^{4\text{He}} + S_L^{3\text{H}} + \delta$, are rather close to its final values obtained. Thus the antisymmetrization effects prove to be small. And at $q \geq 0.9 \text{ fm}^{-1}$ the approximation $S_L^{4\text{He}} + S_L^{3\text{H}}$ of Eq. (6), indeed, proves to be close to the net $S_L^{7\text{Li}}$ sum.

If one considers the correlation term itself the antisymmetrization effects are more pronounced. They increase as q increases. At $q = 0.6 \text{ fm}^{-1}$ the relative deviation of the correlation term calculated without full antisymmetrization, *i.e.* $S_L^{4\text{He}} + S_L^{3\text{H}} - 3 + \delta$, from that calculated with full antisymmetrization is 0.6%. At $q = 0.9$ and 1.1 fm^{-1} it is 6% and 18%, respectively, while at $q = 1.3 \text{ fm}^{-1}$ it reaches already 40%. However, at higher q values the contribution of the correlation term to the sum is small as compared with the $Z = 3$ contribution. Even at lower q values the contributions from terms with permutations in Eq. (10) prove to be rather significant but their effects cancel each other to a large degree. Still, the term without nucleon permutations is the leading one which justifies the improvement of the results done above.

In the q region around 1 fm^{-1} the correlation function of ${}^7\text{Li}$ proves to be close to that of ${}^4\text{He}$. The latter is well represented by the second term in Eq. (9). At lower q values these two correlation functions differ from each other due to correlations between nucleons belonging to different clusters. And at higher q values, where these correlation functions are small, they differ from each other due to antisymmetrization effects.

Let us also comment on the above antisymmetrization procedure. The relative motion function we use was fitted to data in the framework of the cluster model without antisymmetrization between nucleons belonging to different clusters. And behavior of this function in the region of an overlap of the clusters was essential at fitting the data. Our use of this relative motion function in conjunction with the calculation with full antisymmetrization is an approximation. It is still applicable quantitatively as the first approximation at those q values at which the effect of inter-cluster antisymmetrization is small. Such an approximation was used in the literature, *e.g.* in [15] in the ${}^6\text{Li}$ case. To our knowledge, fully antisymmetrized cluster model wave functions of ${}^6, {}^7\text{Li}$ nuclei fitted directly to representative sets of data are not available in the literature. (In Refs. [16, 17], for example, unsuccessful fits of this type with simple relative motion functions are listed.) Note also that the node of the relative motion function in the region of the cluster overlap reduces antisymmetrization effects.

Next, let us pass to the S_L sum in the ${}^6\text{Li}$ case. The

TABLE II: The calculated ${}^6\text{Li}$ longitudinal sum. The notation is similar to that in Table 1 except for the last column where the quantity similar to that in the fourth column in Table 1 is presented.

$q, \text{ fm}^{-1}$	$S_L^{4\text{He}} + 1$	δ	$S_L^{6\text{Li}}$
0.5	4.60	1.91	6.26
0.6	4.45	1.38	5.58
0.7	4.29	0.94	5.00
0.8	4.13	0.60	4.53
0.9	3.97	0.34	4.17
1.0	3.82	0.17	3.89
1.1	3.67	0.055	3.67
1.2	3.54	-0.009	3.51
1.3	3.43	-0.040	3.39
1.4	3.33	-0.051	3.30
1.5	3.24	-0.050	3.23

${}^6\text{Li}$ ground state wave function is taken to be $\mathcal{A}\psi$ where ψ is the product of the α -particle wave function, three-cluster $\alpha + p + n$ relative motion function, and the spin-isospin function of the outer proton and neutron. And \mathcal{A} is the operator of antisymmetrization with respect to all six nucleons. The $\alpha + p + n$ relative motion function is taken from Ref. [18]. Only its main configuration having the weight more than 95% is employed. It is of the form $f(r, \rho)$ where r is the distance between outer nucleons and ρ is the distance between their center of mass and the α -particle. It is represented in [18] as a sum of Gaussians. The correspondent spin-isospin function of outer nucleons has spin equal to one and isospin equal to zero.

The calculation is done in a way similar to the ${}^7\text{Li}$ case above. If antisymmetrization is disregarded then $S_L^{6\text{Li}}$ can be written as $S_L^{4\text{He}} + 1 + \delta$ where the δ contribution comes from correlations between outer nucleons and those belonging to the α -particle. One has

$$\delta = 4[\tilde{G}_E^p(q)]^{-1} F_{\text{el}}^{4\text{He}}(q)[1 + G_E^n(q)/G_E^p(q)] \times w^{-1} \int d\mathbf{r} d\boldsymbol{\rho} f^2(r, \rho) \exp[i\mathbf{q} \cdot (\mathbf{r}/2 + \boldsymbol{\rho})], \quad (14)$$

w being the weight of the $\alpha + p + n$ configuration retained. The corresponding results are presented in the second and third columns of Table 2. The notation is similar to that in Table 1. It is seen that, similar to the ${}^7\text{Li}$ case, the contribution to $S_L^{6\text{Li}}$ of correlations between outer nucleons and those belonging to the α -cluster is relatively small at $q \geq 0.9 \text{ fm}^{-1}$.

Furthermore, the calculation of the $S_L^{6\text{Li}}$ sum is performed taking into account antisymmetrization with respect to all six nucleons. The same $\alpha + p + n$ relative motion function is employed as above. The α -particle wave function is taken the same as in the ${}^7\text{Li}$ case above.

The results are listed in the fourth column of Table 2. They do not differ much from the $S_L^{4\text{He}} + 1 + \delta$ values provided by the no-antisymmetrization approximation. Thus the net effect of antisymmetrization proves to be rather small. And at $q > 0.9 \text{ fm}^{-1}$ the approximation $S_L^{4\text{He}} + 1$ of Eq. (5) is, indeed, close to the total $S_L^{6\text{Li}}$ sum.

The correlation term entering the $S_L^{6\text{Li}}$ sum proves to be rather close to the correlation term calculated without antisymmetrization, especially at lower q values. At higher q values they are closer to each other than in the ${}^7\text{Li}$ case.

The correlation function of ${}^6\text{Li}$ proves to be close to that of ${}^4\text{He}$ when q ranges between 1.0 and 1.5 fm^{-1} .

At the same time, differently to the ${}^7\text{Li}$ case, in the present case the magnitudes of the direct terms in the nominator and denominator in the relation of Eq. (10) type are comparable with the magnitudes of the terms with permutations. Therefore, the improvement of the results of the type done in the ${}^7\text{Li}$ case is not justified. Also the fact that the net effect of antisymmetrization proves to be rather small is curious in view of this feature.

III. COMPARISON BETWEEN CLUSTER MODEL AND EXPERIMENTAL SUM RULES

The measurements have been performed using the beam provided by the linear electron accelerator LUE-300 at KIPT. The primary goal consisted in obtaining the Coulomb energy of the ${}^6\text{Li}$ nucleus [3] which is expressed in terms of the longitudinal sum rule. Later we observed that the inelastic longitudinal sum of ${}^6\text{Li}$ reaches its limiting value at considerably lower momentum transfers than for other nuclei. In relation to this, more detailed measurements have been done in both the ${}^6\text{Li}$ and ${}^7\text{Li}$ nuclei cases [4, 5].

In the last measurements, the beam of electrons with energy ranging from 104 to 259 MeV was employed. Uncertainty in energy ranged from 0.4 % to 0.6 %. The ${}^6\text{Li}$ and ${}^7\text{Li}$ targets contained, respectively, 90.5% and 93.8% of the corresponding isotope. Electrons scattered to the angles from 34.2° to 160° were detected. Their momenta were analyzed with a double focusing spectrometer [19]. Electrons were registered in the focal plane of the spectrometer with an eight channel scintillation-Cherenkov detector [20, 21]. The experimental setup has been described a number of times in the literature, see *e.g.* [22, 23]. A detailed description of the measurements and data processing is presented in Refs. [4, 5, 23, 24].

Our task was to extract longitudinal response functions at fixed q values for getting inelastic sums. Since this required considerable time the results were published separately for various q values. In the present work, additional values of the sums are obtained, see Figs. 1 and 2.

To get the sum rule (3), the integration over ω up to infinity is to be performed in Eq. (2). The $\omega > q$ region of the integrand is kinematically inaccessible in

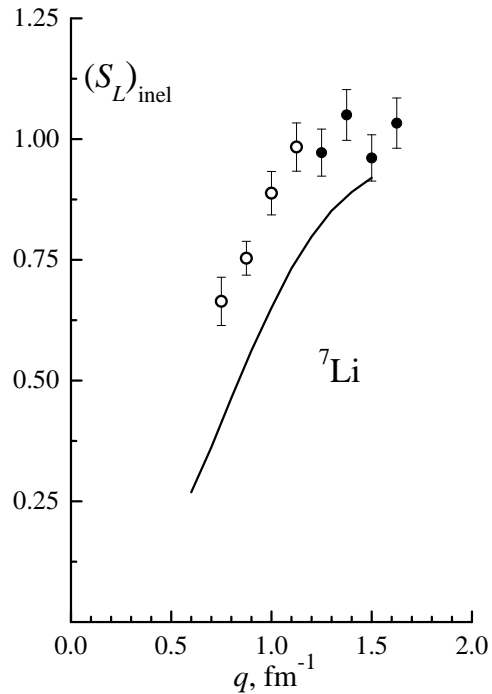


FIG. 1: The inelastic longitudinal sum rule $(S_L)_{\text{inel}}$ of the ${}^7\text{Li}$ nucleus. Experimental points from [5] (closed circles), and from the present work (open circles). Full curve represents the theoretical values in a cluster model, see the text.

experiment. In fact, the experimental response functions $R_L(q, \omega)$ may reliably be determined only at ω values considerably smaller than q . Therefore, they are to be extrapolated beyond these values to get the sums. According to Refs. [25, 26], a reasonable extrapolation of the integrand in Eq. (2) may be realized with a power function $\omega^{-\alpha}$ at $\alpha = 2.5$. Up to experimental uncertainties, the same α value has been obtained in Refs. [4, 5]. In the present work, the same power extrapolation at $\alpha = 2.5$ was performed. The increase in the $(S_L)_{\text{inel}}(q)$ sums due to the extrapolation ranged between 8 and 19 per cent.

Elastic contributions are to be subtracted from theoretical sum rules for comparison with experiment, see Eq. (4). The elastic form factors of ${}^6, {}^7\text{Li}$ nuclei from Ref. [27] are used for this purpose.

In Fig. 1 the inelastic experimental sums $(S_L)_{\text{inel}}$ of the ${}^7\text{Li}$ nucleus are shown along with those calculated in the cluster model. The theoretical values are taken from the column 5 of Table 1 with the elastic contribution subtracted. In addition, a tiny contribution of the 0.47 MeV excited state is subtracted since it is not contained in the data. There is only a qualitative agreement between

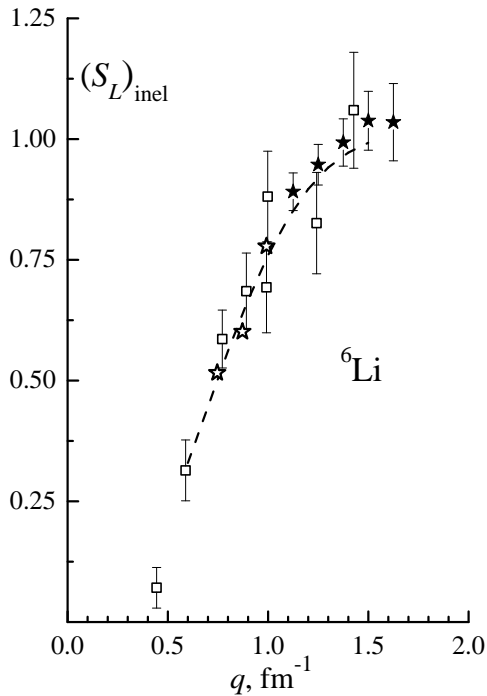


FIG. 2: The inelastic longitudinal sum rule $(S_L)_{\text{inel}}$ of the ${}^6\text{Li}$ nucleus. Experimental points from [3] (squares), [4] (closed stars), and from the present work (open stars without error bars, preliminary). Dashed curve represents the theoretical values in a cluster model, see the text.

experiment and theory here.

In Fig. 2 the experimental $(S_L)_{\text{inel}}$ sums of the ${}^6\text{Li}$ ² As A. Diaz-Torres noted to us, in general such type relations may be of use to establish whether a nucleus is clustered.

nucleus are compared with those calculated in the cluster model. The theoretical values are taken from the column 4 of Table 2 with the elastic contribution subtracted. A complete agreement between experiment and theory is observed.

IV. CONCLUSION

Approximate relations expressing the (e, e') longitudinal sum rules for the ${}^6\text{Li}$ and ${}^7\text{Li}$ nuclei in terms of such a sum rule for the ${}^4\text{He}$ nucleus are suggested.² The $A=6$ and 7 longitudinal sums are calculated in the framework of the cluster models with antisymmetrization done with respect to all the nucleons. It turns out that at momentum transfers higher than 0.8 fm^{-1} the mentioned relations expressing the $A=6$ or 7 sum rule in terms of the $A=4$ sum rule are rather accurate. Thus the $S_L^{{}^4\text{He}}$ sum along with the sizes of ${}^6\text{Li}$ and ${}^7\text{Li}$ basically determines the inelastic sums pertaining to these nuclei. It is shown that in the region of momentum transfers around 1 fm^{-1} the longitudinal correlation functions of the ${}^6\text{Li}$ and ${}^7\text{Li}$ nuclei are close to that of the ${}^4\text{He}$ nucleus. Basing on this, the difference between the q values at which the high- q limit of the inelastic sum rule is reached in the ${}^{6,7}\text{Li}$ cases and the ${}^4\text{He}$ case is explained.

In the present work, the longitudinal sums are obtained in experiment in the range between 0.750 and 1.125 fm^{-1} in the ${}^7\text{Li}$ case and between 0.750 and 1.000 fm^{-1} in the ${}^6\text{Li}$ case. (The ${}^6\text{Li}$ data are preliminary ones.) These experimental sums along with those that were obtained previously at $q \leq 1.625 \text{ fm}^{-1}$ [3–5] are compared with the sum rule calculated in the framework of cluster models. In the ${}^6\text{Li}$ case a complete agreement is found while in the ${}^7\text{Li}$ case an agreement is only at a qualitative level.

V.D.E. thanks S.B. Dubovichenko for useful information.

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The running header: (e, e') sum rules in $^{6,7}\text{Li}$